Volcanic tremor during eruptions: Temporal characteristics, scaling and constraints on conduit size and processes

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A R T I C L E   I N F O

Article history:
Received 1 June 2007
Accepted 7 March 2008
Available online 26 March 2008

Keywords:
volcanic tremor eruptions conduit radius explosions scaling

A B S T R A C T

We investigated characteristics of eruption tremor observed for 24 eruptions at 18 volcanoes based on published reports. In particular, we computed reduced displacements (D R) to normalize the data and examined tremor time histories. We observed: (a) maximum D R is approximately proportional to the square root of the cross sectional area of the vent, however, with lower than expected slope; (b) about one half of the cases show approximately exponential increases in D R at the beginnings of eruptions, on a scale of minutes to hours; (c) one half of the cases show a sustained maximum level of tremor; (d) more than 90% of the cases show approximately exponential decay at the ends of eruptions, also on a scale of minutes to hours; and (e) exponential increases, if they occur, are commonly associated with the first large stage of eruptions. We estimate the radii of the vents using several methods and reconcile the topographic estimates, which are systematically too large, with those obtained from D R itself and theoretical considerations. We compare scaling of tremor D R with that for explosions and find that explosions have large absolute pressures and scale with vent radius squared, whereas tremor consists of pressure fluctuations that have lower amplitudes than the absolute pressure of explosions, and the scaling is different. We explore several methods to determine the appropriate scaling. This characteristic helps us to distinguish the type of eruptions: explosive (Vulcanian or Strombolian) eruptions versus sustained or continuous ash (e.g. Plinian) eruptions. Average eruption discharge, estimated from the total volume of tephra and the total duration of eruption tremor, is well correlated with peak discharge calculated from cross sectional area of the vent and velocity of volcanic ejecta. These results suggest similar scaling between different eruption types and the overall usefulness of monitoring tremor for evaluating volcanic activity.

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1. Introduction

Volcanic tremor that occurs during eruptions (hereinafter termed eruption tremor) is associated with the upward migration of magma and gases through the vent, and includes much information on eruption dynamics and kinematics of magma movement underground. Hence, source processes of eruption tremor are important to understand eruption mechanisms, and monitoring tremor is useful to determine eruption parameters quantitatively. Tremor in general, including eruption tremor, has been documented at more than 160 volcanoes worldwide (McNutt, 1994a,b). In this paper we investigate systematic relations between tremor reduced displacement, a normalized amplitude metric, and factors such as vent radius, erupted volume, and tremor time history with the purpose of deducing general scaling relationships.

The source processes of volcanic tremor, which is not always “eruption tremor”, have been investigated at many volcanoes around the world for several decades (e.g., Aki and Koyanagi, 1981; McNutt, 1986; Mori et al., 1989; Nishimura et al., 1990; Chouet, 1996; Neuberg et al., 2000). As a result, many characteristics of tremor have been identified and quantified. Examples include: ambiguous onset and unclear phases with a predominant frequency of about 1–3 Hz, and an overall frequency range of 0.5–10 Hz; various duration times from a few tens of seconds (so called isolated tremor) to 10 days or more; and hypocenter depths ranging from the surface down to 60 km (summarized in McNutt, 1994b). To explain these features of tremor, especially the predominant frequencies, numerous theoretical models have been proposed. These include resonance of a magma body under the ground (e.g., Crosson and Bame, 1985; Chouet, 1986, 1996), fluid movement in volcanic conduits or channels (e.g., Ferrick et al., 1982; Honda and Yomogida, 1993), bubble growth and collapse in magma or water (e.g., Leet, 1988), and non-linear excitations caused by magma flow (Julian, 1994). However, these proposed models do not always explain well the characteristics of all volcanic tremor, because different types of tremor are observed associated with varying magma properties, vent geometries, volcano structures, and eruption styles.

In the present study, we focus only on eruption tremor to avoid difficulties arising from analysis of tremor from unknown sources. The merits of this approach are as follows: (1) discrimination of eruption
tremor from other types is comparatively simple because eruption tremor is associated with the surface phenomena of eruptions; (2) data for many eruptions are available, with comparatively large seismic signal amplitudes, hence good signal-to-noise ratios; (3) the general physical processes for generating eruption tremor are similar, since all tremor accompanies the common phenomena of eruptions; and (4) measured surface phenomena help us to constrain realistic source processes.

Several detailed studies on eruption tremor have been reported. For example, Eaton et al. (1987) showed that the amplitude of tremor increases with heights of lava fountains at Kilauea Volcano. Yamasato et al. (1988) investigated the temporal characteristics of eruption increases with heights of lava fountains at Kilauea Volcano. Nishimura et al. (1995) inferred that the source mechanism of eruption tremor at Mt. Tokachi can be represented as a counter force of the eruption (single force). Nishimura (1998) extended this work to a suite of explosion earthquakes at 11 volcanoes and determined scaling relationships. From a comparison of tremor at different eruptions, McNutt (1994a, 2004) showed a linear relation between $\log_{10}(\text{reduced displacement})$ ($D_B$) of maximum sustained amplitude of eruption tremor and the Volcanic Explosivity Index (VEI; Newhall and Self, 1982) as:

$$\log_{10}D_B = 0.46 \text{ VEI} + 0.08$$

In this paper, we examine systematic behavior of temporal variations and scaling laws for the amplitude of eruption tremor. To describe the

<table>
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<th>Event no., volcano</th>
<th>Date (start)</th>
<th>Type</th>
<th>$V_1$</th>
<th>Height</th>
<th>VEI</th>
<th>Duration</th>
<th>$D_B$</th>
<th>Freq.</th>
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<td>2</td>
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<td>2</td>
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<td>2</td>
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<td>Power et al. (1994); McNutt (1994a,b)</td>
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$D_B = \text{ maximum reduced displacement}$
average features, we investigate eruption tremor from many eruptions at different volcanoes around the world on the basis of published studies and reports. First, we examine amplitude, estimates of vent size and general characteristics of temporal variations of tremor amplitude. Second, we compare the eruption tremor with explosion earthquakes that accompanied Vulcanian or strong Strombolian eruptions. Subsequently, we examine the discharge rate of volcanic materials from the vent, based on the data of eruption tremor and other geological evidence. Finally, we discuss the source mechanisms of eruption tremor in light of our new interpretations.

2. Observed characteristics of eruption tremor

We investigate 24 examples of eruption tremor at 18 volcanoes on the basis of published reports. In Table 1, parameters of eruption tremor analyzed in the present paper are summarized with information on eruption type, tephra volume, ash column height, and other parameters. Height of ash column and volume of tephra are followed by references. Most of the VEI values are determined by Simkin and Siebert (1994), but some of the VEIs are estimated by us from heights of ash columns and volumes of tephra. For descriptive purposes, we classified eruptions into four types: (1) eruptions from a central circular vent, (2) fissure eruptions, (3) lava lake activity, and (4) phreatic or phreatomagmatic eruptions. Vents are characterized by either radius \( r \) or fissure length \( l \), and the area (cross sectional area) of the vent is calculated from the formula of either \( \pi r^2 \) or \( l \times 1 \text{ m} \). (fissures are assumed to have a thickness of 1 m; this may slightly underestimate some values). We use the maximum fissure length and do not adjust for temporal variations in the portion of the fissure erupting. Radii of vents and fissure lengths are estimated from topographic maps, photographs of eruptions and published reports. Flow velocity \( v \) is estimated by using the relation \( \sqrt{gh} \), where \( h \) is the height of fountaining, ballistics, or plume, and \( g \) is the gravitational acceleration. Because this formulation neglects the momentum of entrained fluid flow, it may slightly underestimate the velocity. We have used the best and highest resolution data available for most of these estimates, however, as in any study using published data (instead of original data), some prudence must be exercised in interpreting the measurements.

We first discuss the amplitude of eruption tremor. Fig. 1 shows a relation between reduced displacement and cross sectional area of the vent as measured at the ground surface. Reduced displacement \( D_k \) (Aki and Koyanagi, 1981; Fehler, 1983) represents a normalized amplitude \( D_k \) (is equal to rms amplitude times distance), which is corrected for geometric spreading and instrument gain. Note that the reduced displacement in the present study is estimated from the maximum amplitude of eruption tremor; distances to stations are given in maps or tables in the various papers cited. Most of the reduced displacements were determined by us and the others were previously determined by McNutt (1994a). We find that the reduced displacement is roughly proportional to the cross sectional area of the vent or fissure, although some of the tremor from fissures shows very large reduced displacements (e.g., events 2, 3 and 8). The slope of the best fit regression line is 0.3, with a regression coefficient of 0.52. This can be written as \( \log(D_k) = \log(0.29 \times \text{cross sectional area}) + 0.52 \). We infer that the maximum reduced displacement is approximately proportional to the square root of the area of vents, that is, the reduced displacement linearly increases with crater radius when vents form a circular crater. This correlation suggests that the area of the vent (crater or conduit) plays an important role in controlling the amplitude of eruption tremor, which is similar to the relation for volcanic explosion earthquakes (Nishimura and Hamaguchi, 1993; Nishimura, 1995, 1998).

Next, we examine temporal variations of eruption tremor amplitude. Fig. 2(a) shows the observed variation of eruption tremor amplitude during the October 1983 eruption of Mt. Miyake, which produced lava fountaining (Uhiro et al., 1984). We see that the amplitude increased abruptly when the eruption started. After reaching a maximum level, the tremor amplitude decreased approximately exponentially, and eventually returned to the noise level. Fig. 2(b) is an example of the November 1964 eruption of Raoul Island that produced a phreatomagmatic explosion (Adams and Dibble, 1966). Like Miyake, the tremor amplitude at Raoul Island increased suddenly as the eruption began, followed by approximately exponential decay (the amplitude also shows a small perturbation in the middle of the decay sequence at about 18 h). Fig. 2(c) shows an example of tremor observed during the June 27, 1992 eruption of Mt. Spurr (McNutt et al., 1995). This case is slightly different from the former two cases. Tremor amplitude showed an approximately exponential increase for about 3.5 h after the eruption started, and reached a maximum. Then, the amplitude decayed approximately exponentially over about 43 m. Scandone and Malone (1985) show the temporal variation of tremor accompanying the 1980 eruptions of Mt. St. Helens (Fig. 2(d)). On May 18, continuous tremor started 3 h after the beginning of the initial gigantic explosion. The amplitude increased approximately exponentially for about 4 h and reached a maximum. After the tremor sustained this maximum level for about 1 h, during which small fluctuations were observed, the amplitude decayed. For the later eruptions at Mt. St. Helens (May 25, June 12, July 22, 1980 (Fig. 2(d)) and later), we find that for all the cases eruption tremor increased abruptly, then decayed approximately exponentially. Note that we have been careful to state the curve shapes as "approximately exponential". This reflects the fact that we have not performed formal curve fitting to all the data. However, several cases for which curve fitting have been done are indeed exponential (e.g. Pavlov – 1996 eruption; J. Benoit, writt. comm.; Shishaldin – 1990 eruption; G. Thompson, writt. comm.). Benoit et al. (2003) showed that scaling relationships between tremor amplitudes and durations for tremor at nine volcanoes were exponential, further supporting this generalization. The implications of this are explained below. In the remainder of the paper, however, we use the terms "gradual increase" and "gradual decrease" with the implicit understanding that these are approximations. In all cases but one the tremor time histories are concave upwards for the increasing and decreasing segments.
A detailed analysis of the events in Table 1 showed three main characteristics in the temporal variation of tremor amplitude: (1) an exponential increase [which we call stage I], (2) maintenance of a maximum level [stage II], and (3) an exponential decrease [stage III]. Durations of the exponential increase and decrease usually differ, with the increase having a longer time constant. We also observe that small fluctuations and perturbations are common during these stages and that some of the tremor has slightly more complicated time histories. However, fluctuations are usually smaller than the variations of the three main characteristics, and some of the complicated shapes (time histories) can be explained by a superposition of several exponential increases and decreases (e.g., tremor on February 4, 1989, at Mt. Tokachi; see Fig. 7 of Nishimura et al., 1990). Therefore, we conclude that an exponential increase, the maintenance of a maximum level, and an exponential decrease are the three most basic stages of eruption tremor. The durations of the three characteristics are displayed for each event in Table 1: 58% of the events clearly show an exponential increase, 58% show the maintenance of a maximum level, and 92% show an exponential decrease.

The occurrence ratio for each stage in this analysis is approximate. Because our data were selected from figures in previously published reports, our classification depends on the resolution of the figures for each eruption. For example, it is difficult to judge whether or not an exponential increase occurred in cases of rapid increase as shown in Fig. 2(a) and (b). The criteria of exponential decrease and maintenance of a maximum level also have similar problems for other tremor episodes. Hence, our classification in Table 1 is judged by which processes are dominant in the sequence of tremor. To evaluate each stage, we measure total duration times for each stage, $\tau_I$, $\tau_{II}$, $\tau_{III}$, which are the times from the start of eruption to peak amplitude, time of fluctuating part, and time from peak until end, respectively. If these three stages are added together, we obtain the total duration of the eruption. Table 1 also shows $\tau_b$ and $\tau_e$, for an exponential increase at the beginning and an exponential decrease at the end of eruption, respectively, which are measured from the peak to 36% of the peak ($=1/e$). These are not durations, but instead are characteristic times for exponential increases or decreases in amplitude. We did not measure the time constants $\tau_b$ and $\tau_e$ for a few examples of tremor that were not matched with the three stages (e.g., concave downward decay of the 1977 eruption of Usu).

We find that the occurrence of an exponential increase (58% of cases) is less likely than that of an exponential decrease (92% of cases), although exponential increases are clearly observed at the May 18 eruption of Mt. St. Helens, at the first eruption of Mt. Spurr, June 27, 1992, and at the November 1964 event of Mt. Shiveluch. As in the 1980 eruption sequence of Mt. St. Helens (Fig. 2(d)), the exponential increase often occurs accompanying the first main eruption, but seems to be less frequently observed before the second and later eruptions. Hence, we infer that the exponential increases are mainly associated with the first paroxysmal phase, or in the early stages of an eruption sequence at a volcano.
The main characteristics of eruption tremor are summarized as follows; (a) the maximum reduced displacement is approximately proportional to the square root of the cross sectional area of the vent, (b) the eruption tremor shows three basic stages: a gradual increase, maintenance of a maximum level, and a gradual decrease, and (c) the gradual increase observed for the first big eruption of a volcano is generally very clear.

3. Comparison of the eruption tremor to explosion earthquakes

Volcano seismologists have classified volcanic explosion earthquakes and eruption tremor mainly based on the following points. Explosion earthquakes produce simple waveforms with short durations of less than a few tens of seconds, often accompanied by an air-shock wave that is superimposed on the seismograms or is recorded by a micro-phone or infrasound meter. On the other hand, eruption tremor generally has a long duration (e.g. minutes to hours) with emergent onsets, and is not accompanied by large air-shock waves. These two types of signals are generally associated with different types of eruptions, such as brief Strombolian or Vulcanian-type explosions and sustained ash or lava emissions, respectively. Hence systematic differences between eruption tremor and explosion earthquakes reflect differences in the dynamics of eruption in these eruption styles.

In Fig. 3 we plotted explosions and tremor magnitudes as functions of the radius on the same axes. Here we covert $D_R$ to magnitude (Tuboi, 1954; Watanabe, 1971; see Nishimura, 1998 for details) so we can use the same base plot as Nishimura (1998). Explosions scale with $r^2$, so the slope is 2 in Fig. 3. Explosions require a high absolute pressure; that is, pressure builds up under a sealed cap and rupture occurs quickly when the pressure exceeds the strength of the cap. For Vulcanian eruptions the cap is solid rock, whereas for Strombolian eruptions the upper slug of magma serves the same purpose. This pressure is estimated to be 1–10 MPa (Nishimura, 1998; Nishimura and Uchida, 2005). Eruption tremor, by contrast, occurs under open vent conditions. Tremor is generated by pressure fluctuations from turbulence within the conduit, and the amplitude of these fluctuations...
must be less than the absolute pressure of explosions. Further, tremor is a sustained signal, so the fluctuating pressures persist for periods of minutes to hours or longer, in contrast to explosions, which have a times scale measured in seconds to minutes. Even though we do not know the exact source mechanisms for eruption tremor, such a difference as can be recognized in Fig. 3 is quite useful to empirically distinguish the styles of eruption: explosive (Vulcanian or Strombolian) or continuous ash emissions (e.g. Plinian). One case of the eruption tremor, for Izu Oshima 1986 (square symbol labelled "3" in Fig. 3), plots almost on the scaling relation for explosion earthquakes. Note that this tremor is twice as strong as that from the 1991 eruption of Pinatubo (tremor point labelled "13" in Fig. 3). Although the Izu Oshima eruption was a basaltic type eruption, this tremor behaved more like an explosion earthquake and not like typical eruption tremor from the view point of seismic wave generation. This determination is also supported by phenomenological observations of a very high eruption column reaching 16 km a.s.l. and by the generation of visible shock waves from the active vent. Additionally the eruption may have broken fresh rock to form a new fissure, which would contribute to stronger tremor by a geometric effect as shown in Fig. 1.

4. Discharge rate of eruptions

Our systematic measurements in Table 1 permit an additional comparison to be made. Fig. 4(a) shows a comparison of two kinds of discharge rates of tephra. The first is the average discharge rate \( Q_a \) estimated by dividing the total tephra volume by the total duration of the eruption estimated from tremor duration. The second is discharge rate \( Q_b \) obtained from the column height (Morton et al., 1956; see also below). In Fig. 4(b), the discharge rate in the vertical axis is calculated as the product of the vent area and the estimated peak flow velocity \( Q_{p,v} \); this is the peak or maximum instantaneous discharge. Note that the units of the horizontal and vertical axes are not the same, the former is kg/s and the latter is m³/s. We find that the two peak discharge rates are highly correlated with the average discharge rate. The flow velocity varies over about one order of magnitude (20–420 m/s) and the vent area across five orders (5 × 10⁴ to 3.1 × 10⁶ m²), therefore, we conclude that cross sectional area is more important parameter in controlling the mass flux. Because tremor amplitude is proportional to the square root of the cross sectional area, we can in principle quantitatively evaluate the discharge rate of eruption by monitoring tremor amplitude. It is noteworthy to mention that \( Q_a \) is strongly proportional to \( Q_{p,v} \), which enables us to roughly evaluate an average discharge rate by measuring only the cross sectional area of the vent and assuming a representative flow velocity.

5. Discussion

We wish to compare quantitatively the differences between tremor and explosion earthquakes, so that we can infer some of the physical factors that govern tremor occurrence during eruptions. To do so we present a straightforward model of tremor generation. First, we suppose that the eruption tremor is generated by pressure fluctuations in a cylindrical conduit due to volcanic flows. The conduit shape is not critical here and a cylindrical conduit is mathematically convenient. We envision that expanding gases and flow of magma push against the wall rocks as magma moves towards the surface to erupt. We then quantitatively represent a source of eruption tremor. Eruption tremor consists mainly of surface waves (McNutt, 1994b), so the source is presumed to be located at a shallow portion of the volcanic conduit. We use a cylindrical conduit with a radius of \( R \) and a length of \( L \) as a source configuration, and the tremor represents radial oscillations of the conduit wall in and out from its neutral position. In this case, the moment tensor of the source, \( M \), is expressed by e.g., Chouet (1996) in the xyz coordinate (\( z \) axis is the vertical direction):

\[
M = \begin{pmatrix}
\lambda + \mu & 0 & 0 \\
0 & \lambda + \mu & 0 \\
0 & 0 & \mu \\
\end{pmatrix} dV.
\]

(2)

where \( \lambda \) and \( \mu \) are the Lame constants, \( dV \) the volume change of the source. We use the far field expression for Rayleigh waves from this seismic moment tensor (e.g., Aki and Richards, 1980; Eq. (7.149)) on page 316) as the displacement of eruption tremor:

\[
[u] = \frac{r_2(0)}{4\pi U_0} \sqrt{\frac{2}{k^2}} \left( 2k r_1(h) + \frac{dr_2}{dh} \right) M_0
\]

(3)

where \( r_2(h) \) is the fundamental mode of the eigen function (we neglect higher modes), \( k \) the phase velocity, \( U \) the group velocity, \( r_1 \) the energy integral, \( k \) the wavenumber, \( L \) the epicentral distance. Here we assume \( \lambda = 0 \) so that \( M_0 = \mu dV \). For simplicity, we assume a semi-infinite medium, so we obtain:

\[
\begin{align*}
& r_1(h) = e^{-0.8475 h} - 0.5773 e^{-0.3933 h} \\
& r_2(z) = 0.8475 e^{-0.8475 z} - 1.4679 e^{-0.3933 z} \\
& h_1 = 1.2049 h/\rho_0 \\
& e = U_0 = 0.0914 f \pi
\end{align*}
\]

(4)

where \( h \) is the source depth, \( \omega \) the angular frequency of tremor, \( \beta \) the S-wave velocity, \( \rho_0 \) the density of medium, and the units are MKS. As a result, the reduced displacement is written as:

\[
D_k = \frac{|u|}{\sqrt{r^2}} = \sqrt{\frac{2r_2(0)}{8\pi U_0}} \left( 1.23 e^{-0.85kh} - 0.58 e^{-0.39kh} \right) M_0
\]

(5)

where \( L \) represents the wavelength. The seismic moment \( M_0 \) can be expressed by using the strain \( c \) in the radial direction at the conduit wall:

\[
M_0 = 2\pi L c R^2 . \epsilon.
\]

(6)

Alternatively, we can express the \( M_0 \) by the pressure disturbance in the flow, \( \Delta P \):

\[
M_0 = 4\pi L c R^2 \Delta P . \epsilon.
\]

(7)

These relations are quite important for estimating the conduit radius \( R \) because the eruption dynamics and the magnitude of eruptions are closely related to the cross sectional area of the conduit. However, Eqs. (5) and (7) indicate that we cannot extract the radius from \( D_k \) without determining \( L \) and \( \epsilon \) (or \( c \)) independently. We can use a fixed \( L \) of 500 m, and assume a maximum value of \( \Delta P \) at 1 MPa to determine a minimum conduit radius under the assumption of cylindrical geometry. Assuming \( \beta = 1.5 \text{ km/s}, \omega = 2\pi \times 2 \text{ rad/s}, h = 250 \text{ m}, \rho = 2500 \text{ kg/m}^3 \), we plot these values versus the radius determined from surface topography in Fig. 5 and list the values in Table 2. For comparison we also determined the radius for spherical geometry (a point source) which is the minimum possible radius that can be determined using seismic data. This is of course physically unreasonable because there is no way for the magma to move, however it provides some insight into the limiting case. These values are quite small and are also shown in Table 2.

We now need to link the conduit radii determined from seismic data to those determined from topography and to use these data to bring together the constraints from both explosions and eruption tremor. Three of our cases have data for both explosions and tremor: Pavlof, St. Helens and Tokachi. These cases are especially useful to determine how much lower the pressure fluctuations are for tremor compared to the absolute pressure of the explosions. In each case the seismic magnitude of the explosions is larger than the tremor by 1–4
orders of magnitude as seen in Fig. 3. Thus the relative sizes of explosions versus tremor appear to be correct in terms of the pressure arguments in Section 3 above; high absolute pressure for explosions and lower pressure for tremor.

We next attempt to reconcile the systematic errors in measuring the vent/conduit radius. We observe in Fig. 3 that the explosion and tremor data have different slopes, which imply fundamentally different scaling relations and different underlying processes. But we must first consider whether the measurements of vent radius (seen at the surface) and conduit radius (what we infer acts to produce tremor at depth) can be improved or corrected. We explored three correction schemes. First, we used Mount St. Helens as a reference value of 50 m because it has constraints based on several different methods (Carey and Sigurdsson, 1985; Chadwick et al., 1988), and shifted tremor data to the left to agree with the same slope as explosions and the observed offset of Mount St. Helens. This scheme is the most restrictive and assumes that the scaling of explosions is correct. Second, we used the conduit radius determined from $D_v$ (cylindrical geometry in Table 2) to rotate the data, again using Mount St. Helens at 50 m as a reference value. This gives the same slope as explosions but preserves the scatter of the data; we presume the scatter has physical meaning. A third scheme, sliding the data to the left but retaining the slope of the tremor data, was rejected because it moves the leftmost points (smallest $R$) unrealistically too far to the left. Of the three methods we prefer method 2 as being best grounded in the observations and also agreeing reasonably well with the theory. However, none of the various correction methods we considered can satisfactorily explain the offset and scatter of tremor data with respect to explosion data without invoking parameters that cannot be directly measured.

We considered other factors that contribute to our understanding and assessment of appropriate values for the vent/conduit radii. For example, plume rise theory based on Morton et al. (1956) suggests that $H=1.67Q^{0.259}$ where $H$ is height and $Q$ is discharge.

Since $Q=\text{velocity} \times \text{cross sectional area}$, for circular conduits this implies $H$ is proportional to $R^{5/2}$. In Fig. 6 we plot $R$ determined from $H$ (using data from Table 1) versus $R$ from $D_v$. We assumed velocity in Table 1 and we used the $R$ determined from $D_v$ with cylindrical geometry (Table 2). We observe that the slope is approximately 1.0 for values of $R$ determined from $H$ greater than $10^9$ in which we consider this to be the range where observations are most reliable. Smaller values give extremely low $R$ estimates (lower part of Fig. 6). We also note that all the values of $R$ based on $H$ are very small. In fact they agree better with the radii determined from $D_v$ using a point source (Table 2; spherical geometry), which is the minimum possible size using seismic data. This suggests again that the main part of the flow during eruptions is concentrated near the center of the conduits so that the effective radius is indeed quite small. The choice of $R$ for various modeling schemes depends very strongly on the conditions of the eruptions and the constraints allowed by the data.

Why would explosions scale differently than volcanic tremor? We suggest that explosions actually form the craters, so the size necessarily scales with the strength of the explosion, assuming that the strength of the rock is constant (Sato and Taniguchi, 1997; Nishimura, 1998). Tremor, on the other hand, apparently occurs associated with sustained eruptions that use only a portion of the available conduit, generally the central part, and does not modify the conduit significantly during the course of the eruption, except perhaps at the vent.

We were surprised to see so little obvious difference between basalt and andesite/dacite composition (different symbols in Fig. 3). We had anticipated an effect because basalt is less viscous and may have a lower gas content, whereas andesite/dacite is more viscous and has higher gas content. The lack of a difference suggests that the physics of sustained explosive eruptions are not very sensitive to magma composition, but depend more strongly on parameters such as conduit size, ascent velocity, etc.

The vent sizes estimated from topographic maps, photographs, etc. are known to be too large, for several reasons. First, the radius is

Fig. 5. Vent size estimated from $D_v$ versus vent size estimated from topography. Numbers and symbols are the same as Fig. 3. Note that radius estimated from $D_v$ is effective radius. See text and Table 2 for details.

![Graph showing vent size estimated from $D_v$ versus vent size estimated from topography. Numbers and symbols are the same as Fig. 3. Note that radius estimated from $D_v$ is effective radius. See text and Table 2 for details.](image-url)
measured at the ground surface, whereas the eruption tremor signal originates at depths of a few hundred meters or more. Geological investigations generally show that volcanic vents are flared at the tops, hence surface values are always maxima. Second, seismological evidence at Mule Creek (Stasiuk et al., 1996) shows breccia, vitrophyre, and degassed magma near the wall rocks, suggesting that the part of the magma that moves, or the effective size, is smaller than the full size measured to the wall rocks. Third, flow models for viscous fluids such as magma (Poiseuille type flow or plug flow) show that the flow velocity is high near the center of a conduit and very slow near the edges, hence the part of the conduit involved in the main magma flow is again smaller than the full conduit dimensions. This implies that only a small part of the flow processes contribute to volcanic tremor generation. This would be the parts near the walls, suggesting that the central maximum flow part is decoupled or does not transmit its energy very efficiently to the adjacent material that is closer to the wall. All these factors likely contribute to the scatter in plots of vent size versus seismic amplitude, and also affect the slope of scaling relations.

Based on the characteristics of eruption tremor, we infer the behavior of the vent and flow as follows. In stage I, either the effective size of vent is enlarged by the flow of volcanic ejecta, or the fluctuation of pressure in the volcanic flow systematically increases, or both. The distribution of gasses in the magma may contribute to the exponential increase, with low gas at first and more gas later, because the upper portion would be partially degassed to the surroundings during slow ascent (e.g. Jaupart, 1998). New magma may gradually be supplied from deeper regions. These factors may increase amplitude of tremor exponentially. In stage II, the cross sectional area of the vent has reached a maximum size and remains roughly constant. The velocity and density of flow do not change because magma supplied to the reservoir and magma withdrawn through the vent are almost equal. Hence, the amplitude of tremor keeps a maximum level. In stage III, the cross sectional area of the vent remains almost at the maximum level, but the velocity of flow decreases as the pressure of the reservoir becomes lower due to cessation of magma supply from deeper reservoirs. As a result, tremor eventually ceases at the end of eruption.

### Table 3

<table>
<thead>
<tr>
<th>Stage</th>
<th>Area size of vent</th>
<th>Velocity and density of flow</th>
<th>Magma supply</th>
<th>Tremor amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Increase</td>
<td>Constant with fluctuation</td>
<td>Yes or no</td>
<td>Exponential Increase</td>
</tr>
<tr>
<td>II</td>
<td>Constant</td>
<td>Constant with fluctuation</td>
<td>Yes</td>
<td>Maintenance of a Maximum Level</td>
</tr>
<tr>
<td>III</td>
<td>Constant</td>
<td>Decrease with fluctuation</td>
<td>No</td>
<td>Exponential Decrease</td>
</tr>
</tbody>
</table>

These changes of the radius, density, flow velocity, etc. for each stage are summarized in Table 3.

### 6. Conclusions

We investigated characteristics of 24 cases of eruption tremor at 18 volcanoes based on published reports. Detailed analyses of reduced displacements and temporal variations of eruption tremor with geological data such as areas of vents, flow velocities of ejecta, and tephra volumes, reveal the following basic characteristics of the tremor: (1) reduced displacement is approximately proportional to the square root of pressure in the volcanic conduit or vent; (2) temporal variation of tremor amplitude shows an exponential increase at the beginning of eruption, followed by maintenance of a maximum level, and exponential decay at the end; (3) exponential increase is often observed at the first big eruption of a sequence.

To explain these characteristics, we investigated scaling relations between tremor amplitude, cross sectional area of the vent (conduit), velocity and density of volcanic flow, and other parameters. From the comparison of these parameters with the characteristics of the eruption tremor, we find that cross sectional area of the conduit or vent is an important parameter controlling the amplitude of tremor and its temporal variation, despite measurement problems. Explosions are stronger than tremor at the same volcano, reflecting the fact that explosions require a high absolute pressure to break the cap rock, whereas tremor consists of lower magnitude fluctuations of pressure in a sustained eruption from an open vent. The observed features of eruption tremor may help us to better understand temporal changes in the magnitude of volcanic eruptions.

### Acknowledgments

We are grateful to H. Hamaguchi for providing us the unpublished data of Volcano Nyiragongo and to K. Ushira for giving us information on tremor data of Mt. Miyake. An earlier draft of the paper was reviewed by J. Benoit, M. Garces, H. Shimozuru and B. Sturtevant. Matthias Hort and Silvio de Angelis kindly provided comments on the current draft. This study was partly supported by the foreign scientist invitation program of the Japan Society for Promoting Science, by the U.S. National Science Foundation under grant EAR-9418219, by the 21COE Program of Tohoku University, and by the U.S. Geological Survey as part of the Volcano Hazards Program, and by additional funds from the State of Alaska to the Alaska Volcano Observatory.

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